In–duct measurements: Eduction of liner impedance

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1.0 INTRODUCTION

This note for the lecture series ”Design and Operation of Aeroacoustic Wind Tunnels for Ground and Air Vehicles” introduces the delicate notion of impedance with flow and the various ways used to try to measure it.

The impedance is a way to characterize the acoustical effect of a locally reacting wall on the acoustical propagation. At first, the impedance without flow is introduced and a link is done between the local and global interpretation of the impedance. The impedance with flow is then introduced.

Thereafter, the different ways for measuring the impedance are exposed. The measurement technics are divided into two large families: the local measurement and the measurements in the far field. In the local approach, one tries to measure the pressure and velocity in the near field. With flow, it is shown that the measure can be perturbed by the hydrodynamic modes. The far field approach is more consistent with the final use of the impedance which is introduced as a boundary condition in the commercial codes that calculate the acoustic propagation. Several measurement approaches are described that give excellent results without flow. With flow, the presented models are not self-consistent and further research is needed to clearly define the notion of impedance with flow.

A much more complete description of the propagation in lined duct can be found in the book [1] freely available on the webpage http://www.win.tue.nl/~sjoerdr/.

2.0 WHAT IS IMPEDANCE ?

2.1 Impedance without flow

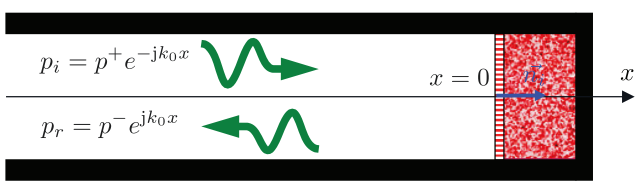

Figure 1: Propagation in a tube.

Let us consider the acoustical propagation in a tube (see Fig. 1) where only plane wave can propagate. If ω is the frequency and the time dependence $\mathrm{e}^{\,j\omega t}$ is omitted, the incident pressure p_i and reflected pressure p_r can be written $p_i = p^+ e^{-jk_0 x}$ and $p_r = p^- e^{\,jk_0 x}$ where $k_0 = \omega/c_0$ and c_0 is the sound velocity. The ratio between the reflected and incident waves at $x = 0$ is given by $R = p_e/p_i = p^-/p^+$ where R is the **reflection coefficient**. At the same point ($x = 0$), the relation between the acoustic pressure, $p = p^+ + p^-$, and the acoustic x-velocity, $v = (p^+ - p^-)/\rho_0 c_0$ where ρ_0 is the density, defines the **impedance** by $Z^* = p/v = \rho_0 c_0 (1 + R)/(1 - R)$. The dimensionless reduced impedance $Z = Z^*/\rho_0 c_0$ will be used in the following of this text.

In this 1D case, the impedance at a given point , here $x = 0$, can be used as a substitute for the acoustic effect of all what happens in $x > 0$. This substitution is only valid for the plane waves propagating in the tube even if some higher modes effect occurs near the boundary $x = 0$. It can be denoted that if the impedance is known at some point x_1 of the tube, it can be computed at another point x_2 by:

$$Z_2 = \frac{Z_1 + \mathrm{j}\tan(k_0(x_2 - x_1))}{1 + \mathrm{j}Z_1 \tan(k_0(x_2 - x_1))} \tag{1}$$

To generalize this notion, we consider the 3-D geometry in Fig. 2 where a volume V is limited by a surface S. The solution can be formally written:

$$p(\vec{x}) = \iiint_V S(\vec{y})\, G(\vec{x}|\vec{y})\, \mathrm{d}^3\vec{y} + \iint_S (G\vec{\nabla}p - p\vec{\nabla}G).\vec{n}\, \mathrm{d}^2\vec{y} \tag{2}$$

where G is the Green's function of the problem and S is the source term. In the most general situation, p and $\partial p/\partial n$ on the surface S depend on the entire acoustic field. However, this is not the case for the class of so-called locally reacting surfaces. The response of such a surface to an acoustic wave is local: $p(\vec{x})$ and $\partial p/\partial n(\vec{x})$ are linearly linked at each point on the surface S and therefore this relation becomes a property of the surface alone. This relation can be written as a Robin's condition (or mixed, or impedance) $\sigma\, p + \partial p/\partial n = 0$ and the surface integral reduce to:

$$\iint_S (G\vec{\nabla}p - p\vec{\nabla}G).\vec{n}\, \mathrm{d}^2\vec{y} = -\iint_S (\sigma G + \frac{\partial G}{\partial n})\, p\, \mathrm{d}^2\vec{y} \tag{3}$$

In this case, the Eq. (2) reduces to an integral equation in p.

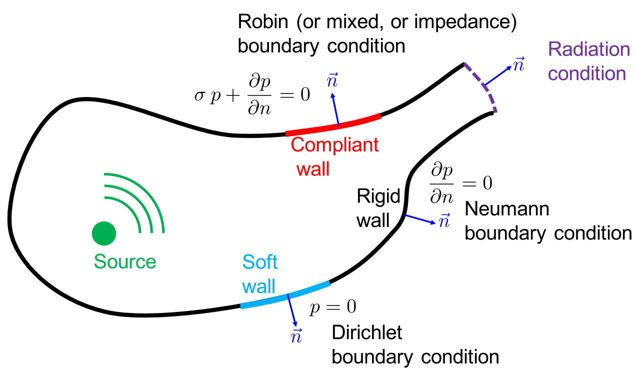

Figure 2: 3-D geometry.

By using the Euler relation along the normal: $\mathrm{j}\rho_0\omega\vec{v}.\vec{n} = -\vec{\nabla}p.\vec{n} = -\partial p/\partial n$ and the impedance definition:

$$\rho_0 c_0\, Z = \frac{p}{\vec{v}.\vec{n}} \tag{4}$$

the coefficient of the Robin's condition is directly related to the impedance by $\sigma = \mathrm{j}k_0/Z$.

The impedance can be seen as the boundary condition that had to be put at a locally reacting wall to have the same effect on the acoustic field that the real wall

In general, the impedance is a complex number $Z = R + \mathrm{j}X$. The real part of the impedance R is called the resistance and is always positive. It is related to the dissipation in the wall. The imaginary part of the impedance X is called the reactance.

2.2 Global vs. local impedance

When a plane normal wave is incident on any material (1D case), we have seen that an impedance can always be defined. In the general 3D case, the material must react locally for an impedance to exist. This concept of locally reacting liner means that the pressure and velocity at any point on the surface are linearly related without influence of the acoustic field at other points of the wall. This implies that there is no acoustic connection between two points of the wall through the interior of the material. One simple example is a material made up of many small closed tubes. Another very classical example is depicted in Fig. 3. This Single Degree of Freedom (SDOF) liner is made of a closed honeycomb structure that prevents the propagation transverse to the wall and of a perforated plate that is used to dissipated energy and to add mass to decrease the resonance frequency. The SDOF can be considered as locally reacting as long as the the wavelength of the incident field is much smaller that the size of the cells of the honeycomb structure.

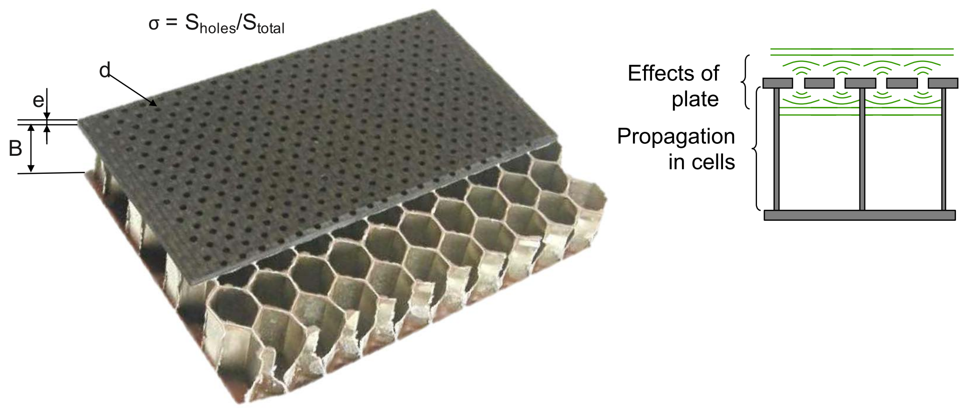

Figure 3: Single Degree of Freedom liner.

Until now the impedance has been defined as a far field quantity connecting the incident to the reflected waves on a boundary. Practically, the wall is very often composed of small periodic or quasi-periodic cells like in the SDOF case shown in Fig. 3. As long as the ration between a length characterizing the periodicity l and the wavelength λ is small $\varepsilon_1 = l/\lambda \ll 1$, it can be shown that the wall can be described by an impedance. For that the field had to be separated in a far field and a near field region and some asymptotic expansion had to be done. The matching of the inner and the outer expansions leads to a definition of the impedance as the ration between the pressure averaged over one cell to the normal velocity also averaged over one cell.

2.3 Impedance with flow

With mean flow the situation is more subtle. When is supposed to be inviscid, there is a slip mean velocity at the wall while the impedance wall is motionless. thus, we had to consider that there exists a (fictitious) vortex sheet, modeling the boundary layer. Since a vortex sheet cannot support a pressure difference, the pressure at the wall is the same as near the wall in the flow. But across this vortex sheet, the normal velocity is discontinuous. The kinematic quantity that is conserved is the normal displacement. In this case, the relation between the pressure and the velocity that can be written in the flow near the wall becomes

$$\rho_0 c_0\, \vec{v}.\vec{n} = (\mathrm{j}\omega + \vec{v}_0.\vec{\nabla})\frac{p}{\mathrm{j}\omega Z} \tag{5}$$

where $\vec{v}_0$ is the mean slip velocity at the wall. This relation valid for a plane wall has been derived by [2] and extended to non-planar geometry by [3] and is called the **Ingard-Myers boundary condition**. Currently, this relation is commonly used in the industry and in the numerical codes which calculate the acoustic propagation with flow [4]. This way of computing the propagation with flow is associated with an impedance Z that changes with the flow and also with the sound pressure level (SPL). Thus the flow effect is included both into the boundary condition (5) and into the impedance Z.

In the academic world, it has been proved that this description can leads to some profound consistency problems [5, 6] and to a disagreement with in-flow measurements [7, 8]. There have been attempts to solve this inconstancy. To that end, some studies relax the assumption of potential flow and use a shear velocity profile [9, 10].

The general idea in the study of the Myers or the modified Myers conditions is to let the thickness of the boundary layer approach zero. This approach solves some of the problems that arise in the " Ingard-Myers + impedance " approach, but not all problems. Especially, the experimental results shows that, even if we consider the shear flow effects and the viscosity effects, the impedances computed for an acoustical wave going in the flow direction and in the direction opposite to the flow are different. This disagreement leads to the question of the actual meaning of an impedance with the flux.

To go from a local description of the phenomena to the far field description, we have seen that a process of homogenization is used without flow. In the flow case, such a rigorous approach has never been made. The equations governing the propagation of sound in a shear flow are much more complex than the Helmholtz equation used in the case without flow. In particular, they involve an infinite number of hydrodynamic modes convected with the flow velocity (which goes from 0 to the maximum value). There at least 3 different small numbers in the shear flow problem: $\varepsilon_1 = l/\lambda$, $\varepsilon_2 = \delta/\lambda$ where δ is the thickness of the mean flow boundary layer and $\varepsilon_3 = \lambda_h/\lambda$ where λ_h is the wavelength of the hydrodynamics modes. Those three small parameter can have the same order of magnitude and in the homogenization process all those contributions has to be taken into account.

It is only at the end of this mathematical process that the question of the existence or not of a simple description (like in the "Ingard-Myers + impedance" approach) will have an answer and that the understanding of impedance with flow will be complete. And so, we can say that, for now, **the notion of impedance with flow is not a well-established concept.**

3.0 HOW TO MEASURE AN IMPEDANCE ?

There are two different ideas to measure the impedance. The first one is directly tied to the local definition of an impedance $\rho_0 c_0\, Z = p/\vec{v}.\vec{n}$ and the pressure p and the normal velocity $\vec{v}.\vec{n}$ are both measured near the impedance wall. The methods using this idea are called **"local method".** The second idea is to measure the effects of the impedance wall on the acoustic propagation. Those methods can be called **"far field method"** or **"eduction method".**

3.1 Local methods

3.1.1 *In-situ* method

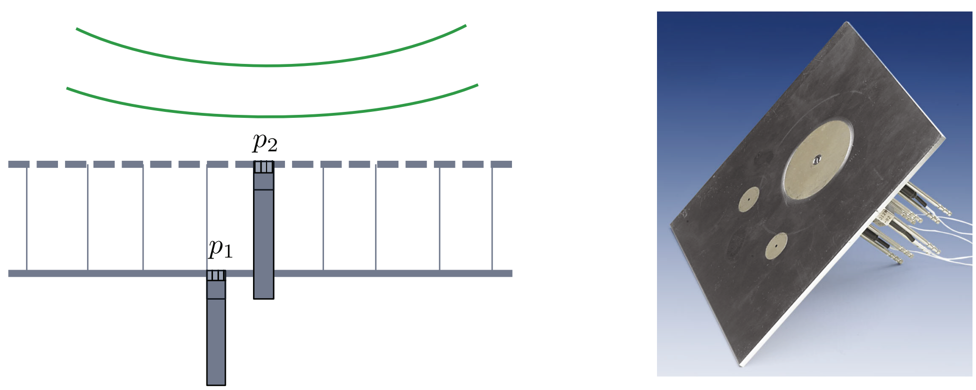

Figure 4: The in situ method.

The *in-situ* method [11] is an example of the local methods that is adapted for SDOF liners. The method uses two microphones, see Fig . 4. The first one is located at the surface of the wall and measure the pressure p. The second one is located at the rigid backing of the liner cavity. In the cavity, the propagation is only transverse to the wall and the 1D acoustic applies. Thus the pressure is written $p = p_1 \cos(k_0 y)$ and the velocity is $\rho_0 c_0 v = p_1\, \mathrm{j} \sin(k_0 y)$. The velocity at the plate level is $\rho_0 c_0 v_L = p_1\, \mathrm{j} \sin(k_0 B)$ and, as the thickness of the plate is suppose to be small compared to the wavelength, this velocity is supposed to be the same than the incident velocity. Thus the impedance can be written:

$$Z_m = \frac{p_2}{\rho_0 c_0 v} = \frac{-\mathrm{j}}{\sin(k_0 B)} \frac{p_2}{p_1} \tag{6}$$

This *in-situ* method is extremely simple to use and can be adapted in various environments: in an impedance tube, in an impedance wall on the side of a duct or of a room and even *in-situ* [12]. This method can be used when the sound level varied and in presence of a grazing flow [13, 14].

However the method is destructive for liner and requires instrumentation in the cells of the liner. With flow this method is rather delicate to apply. The results are very sensitive to the exact position of the surface microphone. This microphone need to be mounted exactly flush to the liner. If it is not the case a small cavity or a small protrusion that can interact with flow and acoustic. Due to this sound-flow interaction in the vicinity of the surface microphone there could be measurement errors in the determination of the impedance.

Another limiting aspect of this *in-situ* method is that the determination of the velocity is only valid for the SDOF liners. If the cavity is divided, like in the two degrees of freedom liners, or filled with some material, the method is much more difficult to apply.

3.1.2 Two outside microphones method

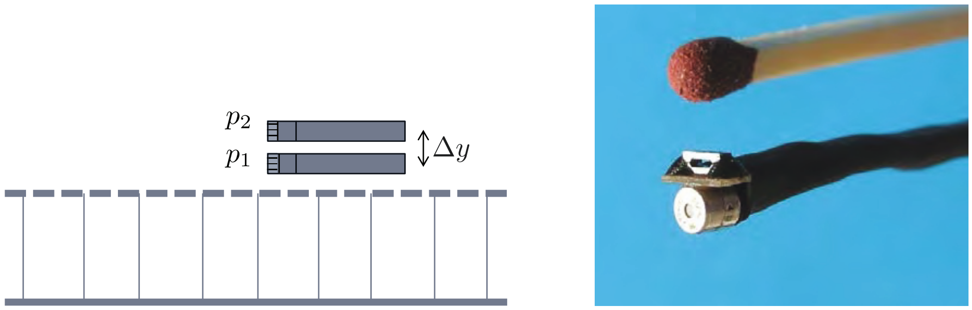

Figure 5: The two microphones method.

An idea to overcome the problem highlighted in the last paragraph of the previous section is to measure the pressure and the normal velocity outside the material. This can be done by two small microphones located in the near field separated by a small distance Δy in the normal direction, see Fig. 5.The normal velocity can be approximated by $\mathrm{j}\omega\rho_0 v = -\partial p/\partial y \simeq (p_2 - p_1)/\Delta y$ and the impedance is given by

$$Z_m = \frac{p_1}{\rho_0 c_0 v} = \frac{\mathrm{j}\omega\Delta y}{c_0}\frac{p_2}{p_2 - p_1}. \tag{7}$$

Another technical possibility is to move a single microphone to two different normal positions. This avoids any problem of calibration between the two microphones. It is also possible to use a pressure-velocity probe (see picture in Fig. 5) which measures in the vicinity of the wall both the velocity and the pressure [15]. All these methods have the advantage that the sample is not deteriorated by impedance measurement, but they are very difficult to apply when a flow is present. All these probes affect the distribution of the flow near the wall of the impedance but above all the pressure or velocity measurements are degraded and in some cases unusable due to the flow.

3.1.3 Transverse probe method

To better understand the flow effects and try to combine the advantages of the two above methods, a transverse probe can be used, see Fig. 6. A microphonic probe is placed on a motorized micrometric displacement system. The probe crosses the cavity and the duct, measuring the acoustic pressure (by two side holes) from the rear wall of the cavity to the opposite side of the duct [16].

An example of pressure profile without flow is given in red on Fig. 6. From these measurements, the standing wave pattern the cavity is verified (of the form $\cos(k_0 y)$) and pressure field in the duct is also known. Figure 6 presents several pressure profiles for different Mach numbers. The pattern in the cavity is unchanged by flow (and so the normal velocity). It can be observed that the pressure jump across the layer varies with flow velocity, showing an influence of the flow on the impedance. In the vicinity of the wall, the flows induces a strong pressure gradient especially on the imaginary part. Therefore, the impedance deduction, which is linked to the choice of a measurement point near layer surface, is a delicate problem.

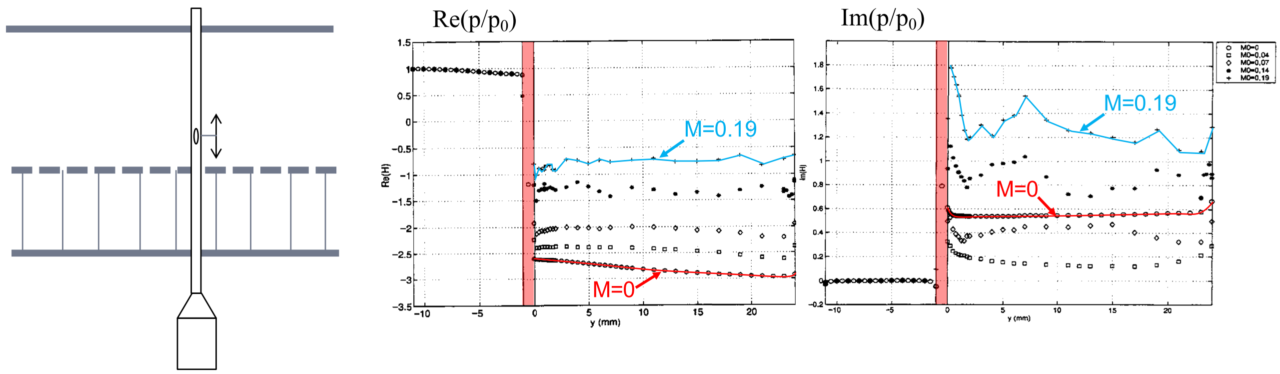

Figure 6: The transverse probe method.

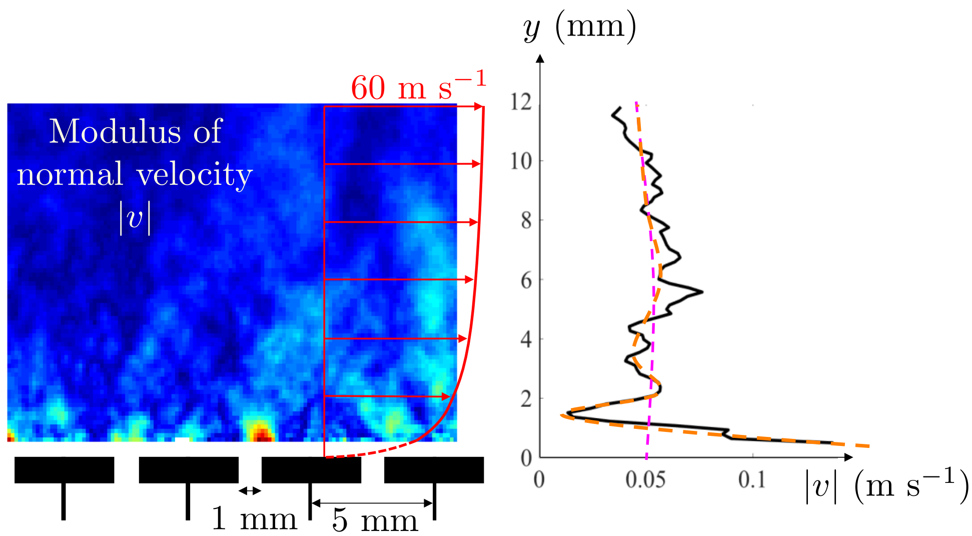

Figure 7: PIV measurements over a perforated liner with flow.

3.1.4 Optical measurements

A fairly new possibility for measuring the acoustic near-field is to use optical measurements. Two main technics are available. The first one is the Laser Doppler Velocimeter (LDV). The flow is seeded by smoke or aerosol particles, and a signal is recorded by the LDV system when a particle crosses the laser fringes. This can be used to measure the longitudinal (x-wise) and vertical (y-wise) velocity components above the liner. The "acoustic" velocity signal is defined as the component of the LDV signal that is correlated with the loudspeaker signal. Finally, amplitude and phase of the longitudinal and normal acoustic velocity components are obtained for each frequency of the excitation signal [17]. The second optical technic is the Particle Image Velocimetry (PIV). The flow is seeded by particles. A laser sheet (perpendicular to the liner) illuminates the particles wile a camera takes two images separated by a small time lag. By correlating the two images one can deduce the displacement of the particles, and then the particle velocity by dividing the estimated displacement by the time lag between the two images. An advantage of this technique is that it makes possible to obtain an instantaneous velocity map over some region of the flow (as opposed to point measurements in the LDV technic) but the PIV technics is less precise on each point [18, 19].

As an example, the normal velocity near a perforated liner submitted to a grazing flow (Mach number $\simeq$ 0.18) is given in Fig. 7. It can be seen in the left part of the figure, that the normal "acoustic" velocity, averaged longitudinally over two cells, is not constant whereas the dimensions are small in front of the wavelength ($\lambda \simeq$ 400 mm). Therefore, one may wonder why there is a large variation of $|v|$ close to the liner. This effect may be related to the existence of an hydrodynamic mode near the wall. In this case, is this mode described in far field propagation or its effect must be included in the definition of impedance with flow?

The near field (or local) measurements of the pressure and of the velocity near an impedance wall are very interesting. Without flow, this kind of measurement makes it possible to measure without ambiguity the impedance of a wall. With flow, the situation is more complex. **Local measurements are a very useful research tool** to better understand the complex interactions between sound and flow that occur in the flow boundary layer. But the impedance values resulting from local measurements must be taken with caution and with care.

3.2 Eduction approach

3.2.1 Impedance tube

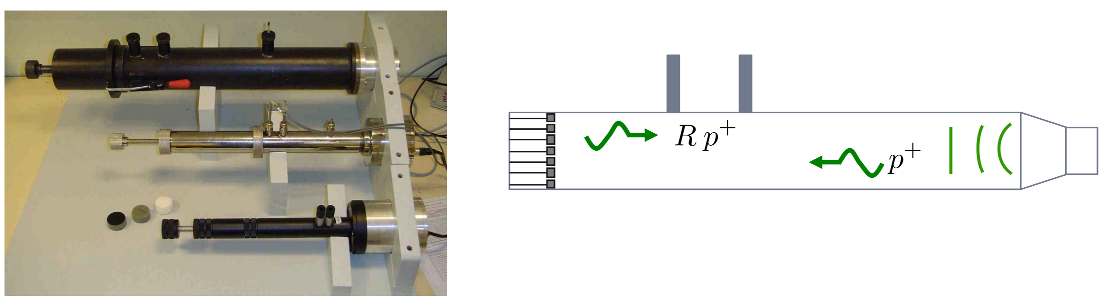

Figure 8: Impedance tube.

The simplest device to measure the effect of an impedance wall on an acoustical wave is the impedance tube, see Fig. 8. In a tube, the acoustic problem becomes 1D in a certain bandwidth where only the plane wave can propagate. We return to the case described in Fig. 1 where the reflection coefficient can be measured with the help of two (or more) microphones. This method is the reference method for measuring impedance without flow and had be normalized [20].

Some portable impedance tubes exist for the measurements of the acoustic properties of materials without having to remove a material sample, see Fig. 9. It can measure objects in situ and is used for verification after installation of acoustic treatments and for control over time.

3.2.2 Eduction technics

The general idea of the eduction technic for measuring impedance is to infer the value of impedance from its effect on the far field. This effect on the far field can measured by dedicated experiences. Those far field can be measured by an array of microphones (for acoustic pressure), by optical methods (for acoustic velocity) or by any combination of these quantities. In general the liner is located in the wall of a duct and is surrounding by rigid ducts. This particular geometry is considered because the main interest of the eduction technic is to

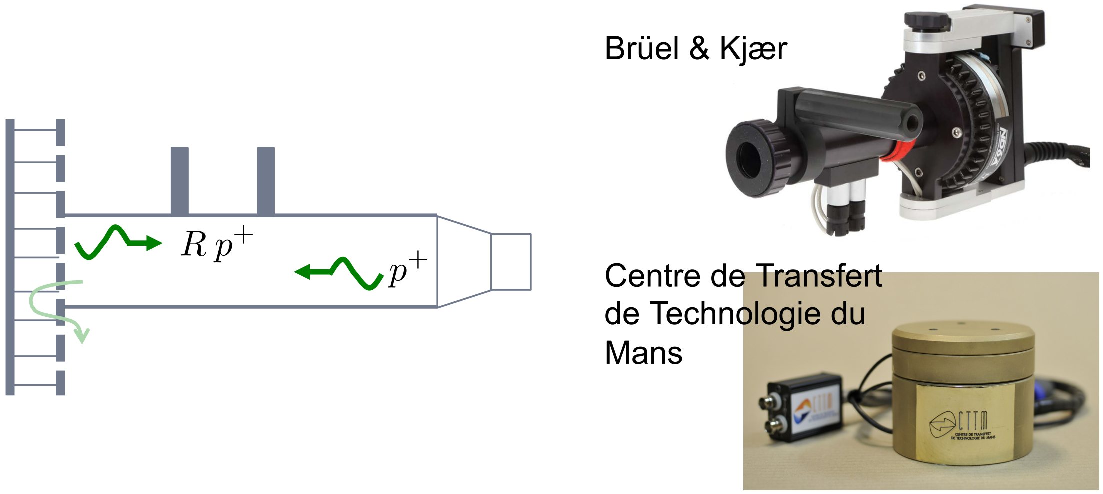

Figure 9: Portable impedance meter and its technical realization by Brüel & Kjær and a more compact device by CTTM (Centre de Transfert de Technologie du Mans).

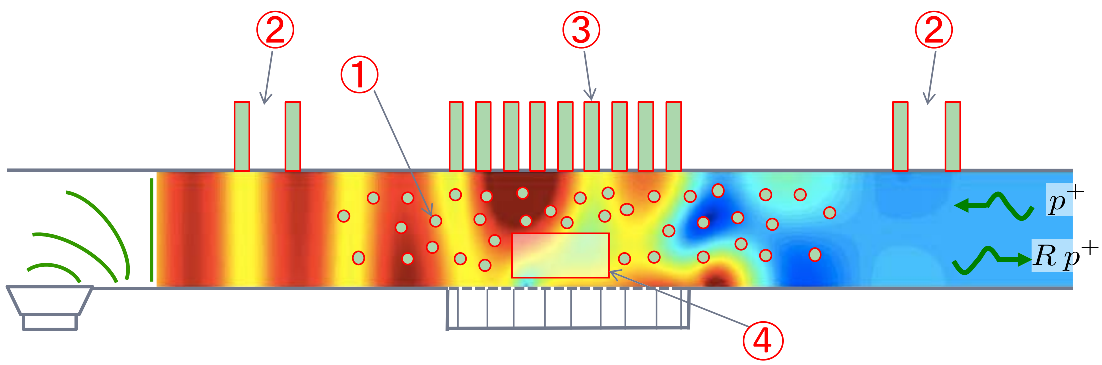

Figure 10: Eduction technics. ①: random array of microphones, ②: array of microphones outside of the lined part, ③: array of regularly spaced microphones in the lined part, ④: window of optical measurements in the lined part

measure the effect of a grazing flow on the value of the impedance and the geometry presented in Fig. 10 allows to put a mean flow in the partially lined duct.

It is supposed that the direct problem can be solved. In this case, we can compute any ratio between two measured quantities (this ration will be called H_i in the following) as a function of the unknown impedance Z and of the reflection of the duct called R in Fig. 10. If several modes can propagate in the tube, the unknowns contain as many reflection coefficients as the number of propagative modes. For simplicity, in the following we will only consider the case where only one mode can propagate in the rigid ducts.

The impedance eduction process consists in solving an inverse problem of parameters identification (here Z and R), by minimizing an objective function $\mathcal{J}$ representative of a distance between a numerical simulation giving the computed quantities H_i^c and the measured quantities H_i^m. This objective function can be written under a general form:

$$\mathcal{J} = \sum_{i=1}^{N} a_i \left| H_i^m - H_i^c \right|^2 \tag{8}$$

where N is the number of measurements which is in general much larger than 2, the number of unknowns, and a_i is a positive weighting coefficient. The minimization of the objective function $\mathcal{J}$ is made by an iterative process with one of the numerous available minimization algorithm [21–26]

This impedance eduction process, as all the indirect method, give at the same time a value of the searched quantities and an idea of the correctness of the model used. Several calculation method can be used to compute the sound propagation in the straight lined duct. The simplest method is to consider an uniform mean flow. In this case, the convected wave equation can be used associated with the Ingard-Myers condition over the liner. To take into account the shear flow, the linearized Euler equations can be used [27]. A step forward can be done by taking into account the viscosity and the turbulent viscosity and solving the linearized Navier-Stokes equations [28]. The next step is currently inaccessible, due to the too long computational time required, and should consist in using a compressible direct numerical simulation (DNS) or a compressible large eddies simulation (LES).

To illustrate the impedance eduction process, we will consider two special cases. The first one is the case numbered ② in the Fig. 10 where all the microphones are outside of the lined zone. From these pressure measurements, we can extract 4 complex numbers that are the coefficient of the scattering matrix and which only depend on the impedance Z. Then, a minimization procedure is applied to extract the value of impedance. In the second example, numbered ③ in the Fig. 10, the microphones are regularly spaced in the lined part and a special procedure is applied to extract the wavenumber of the less attenuated mode over the liner. The impedance is the deduced from this value.

3.2.3 Scattering matrix method

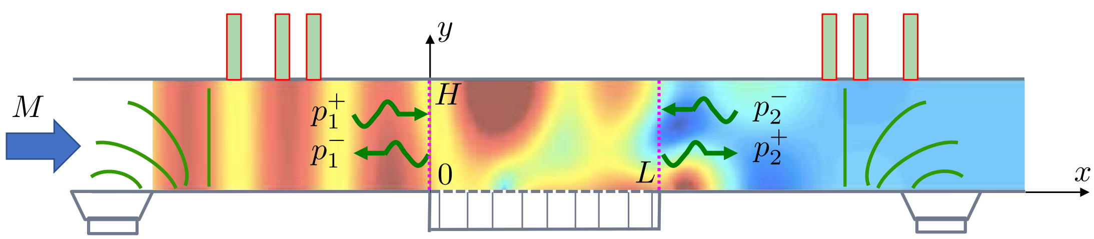

Figure 11: Sketch of the plane wave method

We consider that only plane waves can propagate in the rigid ducts ($x < 0$ and $x > L$) in a setup describe in Fig. 10. Whatever the number of microphones of both sides of the liner, we can only measure p^+ (in flow direction) and p^- on both side of the liner. Thus, the more general description of what happens in the lined part is the scattering matrix for the plane waves S relates the scattered pressure amplitudes p_2^+ and p_1^- to the incident pressure amplitudes p_1^+ and p_2^- by

$$\begin{pmatrix} p_2^+ \\ p_1^- \end{pmatrix} = \begin{bmatrix} T^+ & R^- \\ R^+ & T^- \end{bmatrix} \begin{pmatrix} p_1^+ \\ p_2^- \end{pmatrix} \tag{9}$$

where T^+ and T^- are the anechoic transmission coefficients, R^+ and R^- are the anechoic reflection coefficients, and the subscripts $i = 1, 2$ indicate the upstream and the downstream side of the lined part respectively and the superscripts $\pm$ indicate the direction of propagation along the x axis. The paper [29] reviews the ways of measuring these matrices.

The commonly used method with flow is called ‘the 2 sources method”. Two measurements are made in two different states of the system. These different states are obtained by switching on the upstream source, the downstream source being switched off (measurement I), and then by switching on the downstream source, the upstream source being switched off (measurement II).

When the two measurements are done, the scattering matrix can be obtained with

$$\begin{bmatrix} \left(\frac{p_1^-}{p_1^+}\right)^I & \left(\frac{p_1^-}{p_2^-}\right)^{II} \\ \left(\frac{p_2^+}{p_1^+}\right)^I & \left(\frac{p_2^+}{p_2^-}\right)^{II} \end{bmatrix} = \mathsf{S}^* \begin{bmatrix} 1 & \left(\frac{p_1^+}{p_2^-}\right)^{II} \\ \left(\frac{p_2^-}{p_1^+}\right)^I & 1 \end{bmatrix} \tag{10}$$

if the determinant of the right hand side matrix does not vanish (the superscripts I and II indicate that the quantity had be determined during the measurements I and II). This condition $(p_2^-/p_1^+)^I \neq (p_1^+/p_2^-)^{II}$ is the condition of independence of the two measurements.

The coefficients of the matrix in Eq. (10) can be found from the transfer functions between the different microphones by a relation of the type:

$$(p_1^-/p_1^+)^I = \frac{H^I_{u_j u_i} e^{-jk^+ x_{u_i}} - e^{-jk^+ x_{u_j}}}{e^{jk^- x_{u_i}} - H^I_{u_j u_i} e^{jk^- x_{u_j}}} \tag{11}$$

where $H^I_{u_j u_i}$ is the transfer function between the microphones u_j and u_i obtains in the measurement I, k^+ and k^- are the wavenumbers in the duct in the direction of the flow and in the reverse direction and x_{u_i} is the position of the microphone u_i relatively to the inlet of the measured element. All the other matrix elements can be found on the same way. The wavenumbers k^+ and k^- have to be known to calculate the scattering matrix.

Thus, at the end of this process, the maximum information that can be obtained by scanning the lined part with plane waves consists in 4 coefficients (T^+; T^-, R^+ and R^-) that depend only of the geometry and of the flow (that are known) and of the impedance Z that is the only unknown. An optimization procedure can be used to extract the impedance. First we have to define the objective function:

$$\mathcal{J} = a_1 \left|T_m^+ - T_c^+\right|^2 + a_2 \left|T_m^- - T_c^-\right|^2 + a_3 \left|R_m^+ - R_c^+\right|^2 + a_4 \left|R_m^- - R_c^-\right|^2 \tag{12}$$

Playing with the weighing coefficients a_i, we can distinguish between a wave coming from the upstream side ($a_1 = a_3 = 1$ and $a_2 = a_4 = 0$, the impedance educed from this weighting is called Z^+) and a wave coming from the downstream side ($a_1 = a_3 = 0$ and $a_2 = a_4 = 1$, the impedance educed from this weighting is called Z^-). For the model to be consistent we need to have $Z = Z^+ = Z^-$. The minimization can be made made using the Nelder-Mead simplex method. The choice of the starting point for the minimization is important. With flow the starting admittance is the admittance without flow. As it has been previously mention the quality of the inversion will depend not only of the quality of the experimental results but also of the quality of the model used to compute the scattering matrix as a function of the impedance Z.

For the results of Fig. 12, the flow (Mach number M=0.2) is supposed to be uniform and the convected wave equation is used associated with the Ingard-Myers condition. It can be seen that the two educed impedances Z^+ and Z^- differ showing some inconsistencies of the used model. This model will be improved in the next section by taking into account the effect of the shear in the flow.

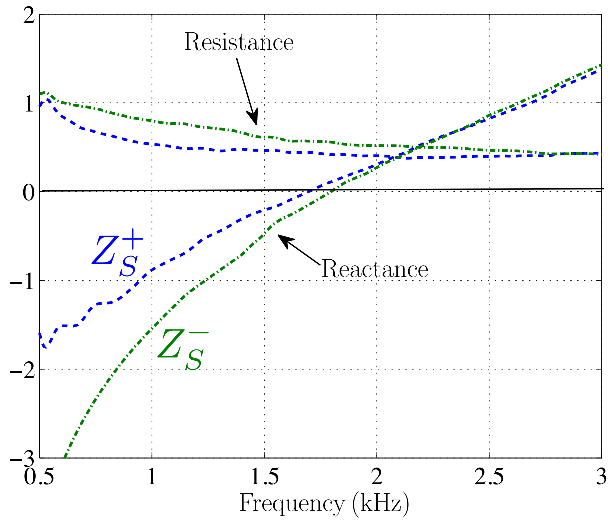

Figure 12: Impedance educed from scattering matrix method for two directions of the incident waves for M=0.2

3.2.4 Wavenumber method

The wavenumbers in a lined section can be computed by various methods (spectral methods, finite difference or finite elements methods). They are costless that the propagation in a lined section because a 2D problem had to be solved rather than a 3D problem. That is why it is interesting to measure the propagation in the lined section and try to deduce the wavenumbers. Then the inverse problem consist only in comparing the measured wavenumbers to the computed wavenumbers which rather easy to do. To measure those wavenumbers, the easiest way is to measure a quantity (pressure or velocity) on regularly spaced point in a line parallel to the liner. In the present applications, this is made by putting a large number of microphones in the wall opposite to the liner sample, see Fig. 13 [7, 30], but we can also imagine to measure the velocity along a longitudinal line by LDV.

Because of the modal expansion that can be used in the lined part when the liner is uniform, we assume that the acoustic pressure measured at each measurement point M_i may be written as:

$$p_i = \sum_{n=1}^{M} \hat{a}_n \left(\mathrm{e}^{-\mathrm{j}k_n \Delta x}\right)^{(i-1)} \tag{13}$$

where M is the number of mode that is taken into account in the propagation, i is related to the abscise of the measured points ($x_i = (i-1)\,\Delta x$) and varying from 1 to the number of measured points N and Δx is the spacing between the measured points.

The analyses of complex exponentials such as Eq. (13) can be done by Prony-like methods. Each exponential $\mathrm{e}^{-\mathrm{j}k_n \Delta x}$ in Eq. (13) corresponds to a pole of the z-transform of p_i. There are methods, like the Kumaresan and Tufts (KT) method that are able to find M poles from the p_i where $i = 1, ..., N$. For detail on the KT method please refer to [7]. When the poles z_p are found it is straightforward to deduce the wavenumbers with $z_p = \mathrm{e}^{-\mathrm{j}k_n \Delta x}$. The number of wave number that can been extracted from the KT method is generally weak: 1 or 2. This method is limited in low frequency by the length L of the lined part which should be long enough to

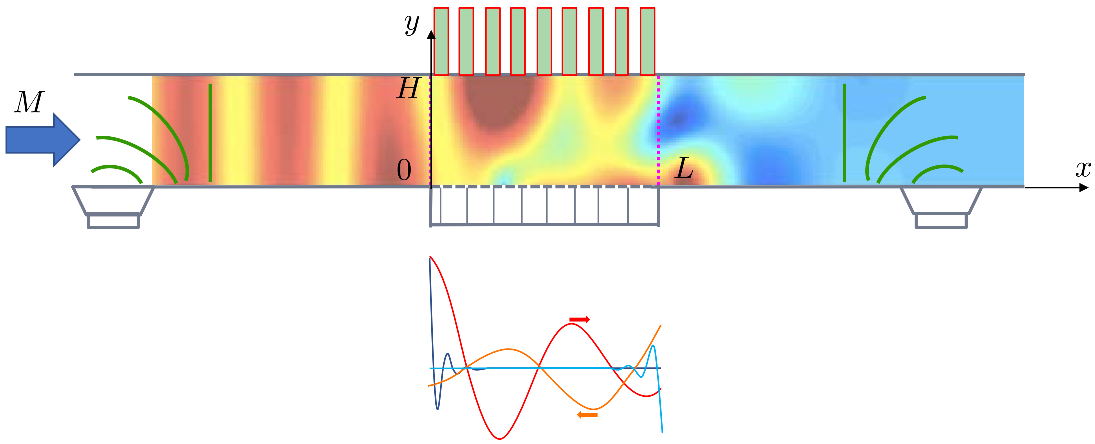

Figure 13: Sketch of the wavenumber method

see significant variation of the amplitude and of the phase of the pressure and is limited in high frequency by the spacing between microphones that must not be much larger that the wavelength.

When the values of the wavenumbers are available, we have to find the value of the impedance by using a model. To go a step further that in the previous example, we will use a shear profile and the Mach number can be written $M(y) = M_0 f(y)$ where M_0 is the mean Mach number and the profile $f(y)$ is found experimentally. Thus we start from the dimensionless Pridmore-Brown equation ($\check{y} = y/H$, $\check{v} = v/c_0$, $\check{p} = p/\rho_0 c_0^2$, $\check{\omega} = \omega H/c_0$, $\check{k} = kH$) that is written:

$$(1 - M_0^2 f^2)\check{k}^2 \check{P} + 2\check{\omega} M_0 f \check{k} \check{P} - \check{\omega}^2 \check{P} - \frac{\mathrm{d}^2 \check{P}}{\mathrm{d}\check{y}^2} = -2\mathrm{j} M_0 \frac{\mathrm{d}f}{\mathrm{d}\check{y}} \check{k} \check{V}, \tag{14}$$

Where the pressure and the transverse velocity has be written $p(x, y) = P(y)\mathrm{e}^{-\mathrm{j}kx}$and $v(x, y) = V(y)\mathrm{e}^{-\mathrm{j}kx}$. The boundary condition at the lined wall, where the mean velocity vanishes, is $\check{P}(0) = Z_{KT}\check{V}(0)$. To solve this problem when the wavenumber k is known, the new variable Y such as $\mathrm{d}\check{P}/\mathrm{d}\check{y} = Y\check{P}$ is introduced. The variation of Y is given by:

$$\frac{\mathrm{d}Y}{\mathrm{d}\check{y}} = -Y^2 - \frac{2M_0\check{k}}{\check{\omega} - M_0 f\check{k}} \frac{\mathrm{d}f}{\mathrm{d}\check{y}} Y - \left(\check{\omega} - M_0\check{k}f\right)^2 + \check{k}^2. \tag{15}$$

This equation can be integrated, by means of a fourth-order Runge-Kutta scheme, from the rigid wall ($\check{y} = 1$) where $Y(0) = 0$ to the impedance wall ($\check{y} = 0$) [6]. This integration process is illustrated in Fig. 13 where the result is compared the solution using uniform flow and the Ingard-Myers boundary condition that give a discontinuity near the impedance wall. It can be noticed that the final results are rather close even if the procedure is quite different.

The educed liner impedance is obtained from $Z_{KT} = -\mathrm{j}\check{\omega}/Y(0)$ and is plotted in the case of a microperforated plate with a Mach number = 0.2 in Fig. 15.

It can be seen that the use of a more precise model does not decrease the difference between the impedance deduced from a wave in the flow direction and from a wave propagating against the flow. On the contrary, the difference is increased when a realistic flow profile is taken into account. Thus, the conclusion is that the impedance equivalent to the wall effect with flow depends on the direction of the incident waves. This discrepancy has been attributed to a non-complete description of the propagation in a shear flow. But the last

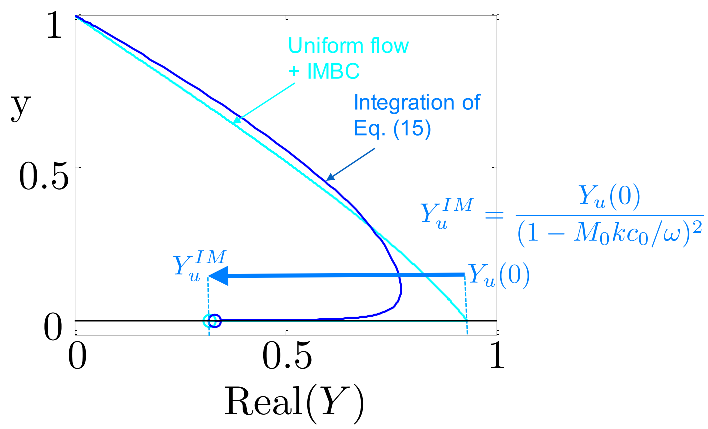

Figure 14: Integration of Eq. (15) in blue and comparison solution using uniform flow and the Ingard-Myers Boundary Condition

example demonstrates that the dependence of impedance on the direction of incident waves is due to a more fundamental issue because the propagation in the shear flow is, here, completely described.

A modeling effort is still needed on the description of liners with flow. The presence of the hydrodynamic modes, the wavelength of which can be small compared to the periodicity of the perforation, is one of the potential issues in the homogenization process of finding a local impedance.

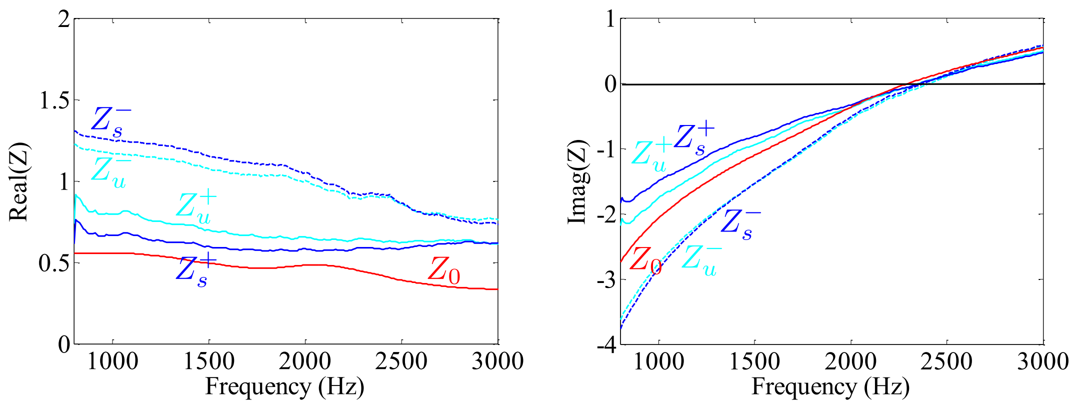

Figure 15: Educed impedance of a microperforated plate. In red the value without flow Z_0. In cyan, the values computed using an uniform flow for both wave directions Z_u^+ and Z_u^-. In blue, the values computed using a shear flow for both wave directions Z_s^+ and Z_s^-.

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